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Estimated eddy diffusivities of momentum and heat of viscoelastic fluids

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Abstract—The eddy diffusivities of momentum and heat for a viscoelastic fluid are evaluated using detailed experimental pressure drop and heat transfer results. The eddy diffusivities of momentum and heat and the turbulent Prandtl number near the wall region are found to be strong functions of the Weissenberg number (i.e. a dimensionless measure of the fluid elasticity) and weak functions of the Reynolds number. The turbulent Prandtl number, $\varepsilon_M/\varepsilon_H$, remains relatively constant for low values of Weissenberg numbers and increases with increasing elasticity up to a maximum value corresponding to the critical Weissenberg number for heat transfer. The maximum value of the turbulent Prandtl number weakly depends on y^+ and the Reynolds number and is found to be of the order of 10.

1. INTRODUCTION

IT IS well known that the addition of small amounts of certain high molecular weight polymers to a solvent results in a viscoelastic fluid. The friction factors and heat transfer coefficients of a viscoelastic fluid in turbulent pipe flow are generally lower than those of the solvent alone when compared at the same Reynolds number [1, 2]. The pressure drop and heat transfer performance of viscoelastic fluids are known to be influenced by a number of factors including the chemistry of the solute and the solvent, the polymer concentration and the level of mechanical degradation. Recent experimental studies [3–5] indicate that the friction factor and the dimensionless heat transfer coefficient may be functions only of the Reynolds and Weissenberg numbers for fully established hydrodynamic and thermal conditions. This implies that the influence of the above-mentioned factors on the fully established turbulent friction and heat transfer behavior of viscoelastic fluids can be successfully accounted for in the Reynolds and Weissenberg numbers.

Another important finding is that critical Weissenberg numbers exist for friction and heat transfer. The critical Weissenberg number for friction is of the order of 10 and that for heat transfer is of the order of 100 where the Weissenberg number is based on characteristic time evaluated by the Powell–Eyring model [6]. Above the corresponding critical Weissenberg number, the fully established friction factor and dimensionless heat transfer coefficient are at their minimum asymptotic value and are functions only of the Reynolds number.

The availability of experimental friction factor and heat transfer data for the turbulent pipe flow of viscoelastic fluids covering a wide range of the relevant variables provides an opportunity for constructing an analytical model. Past experience in dealing with Newtonian fluids reveals that a satisfactory approach to the prediction of turbulent heat transfer is the use of eddy diffusivities of momentum and heat. There have been prior attempts to apply the eddy diffusivity model to viscoelastic fluids but these generally assumed that the eddy diffusivities of heat and momentum are equivalent. There is clear evidence that this assumption is erroneous, especially near the wall region, with the error increasing with the concentration of the polymer [7–9].

In the current study the eddy diffusivity of heat for viscoelastic fluids was evaluated using the experimental heat transfer results, while the friction factor measurements were used to estimate the eddy diffusivity of momentum. The eddy diffusivity of momentum and the velocity profile in the viscous sublayer and the transition region were calculated using a modification of the model proposed by Wasan *et al.* [10] since this model gives a continuous eddy diffusivity of momentum in both regions. To evaluate the eddy diffusivity of heat independently from that of momentum a generalized relationship based on the measurements of Mizushina and Usui [11] was used in conjunction with the energy equation. Under the assumption that the predicted heat transfer results should agree with the measured results it is possible to determine the detailed distribution of the eddy diffusivity of heat.

The final result is an analytical model for the fully established turbulent pipe flow of viscoelastic fluids including a specification of the velocity profile and eddy diffusivities of heat and momentum.

2. ANALYSIS

2.1. *Governing equation*

For steady state, fully hydrodynamically developed turbulent pipe flow with a constant heat flux boundary condition, the governing energy equation and the

NOMENCLATURE

A, B	coefficients defined by equation (12)
c_1, c_2	coefficients defined by equation (10)
c_p	specific heat of fluid
d	diameter of tube
f	Fanning friction factor, $\tau_w/(\rho V^2/2)$
h	convective heat transfer coefficient
j_H	dimensionless heat transfer coefficient, $St\, Pr^{2/3}$
K	coefficient defined by equation (13)
k_l	thermal conductivity of fluid
l	total tube length
m	power law index defined by equation (13)
Nu	Nusselt number, hd/k_l
Pr	Prandtl number based on the viscosity at the wall, $\eta c_p/k_l$
Pr_t	turbulent Prandtl number, $\varepsilon_M/\varepsilon_H$
q''_w	heat flux at the wall
r	radial coordinate
R	radius of tube
Re	Reynolds number based on the viscosity at the wall, $\rho V d/\eta$
St	Stanton number, $Nu/(Re\, Pr)$
T	temperature
T_{in}	inlet temperature
u	axial velocity
u^*	friction velocity, $\sqrt{(\tau_w/\rho)}$
V	average velocity
Ws	Weissenberg number, $\lambda V/d$
Ws_{cf}	critical Weissenberg number for friction
Ws_{ch}	critical Weissenberg number for heat transfer
x	axial coordinate
y	radial distance from the tube wall, $R-r$
y_c	outer edge of the transition zone.

Greek symbols

α	thermal diffusivity of fluid, $k_l/\rho c_p$
Δ	thermal boundary layer thickness
$\varepsilon_H, \varepsilon_M$	eddy diffusivity of heat and momentum, respectively
η	apparent viscosity evaluated at the wall
ν	kinematic viscosity
λ	characteristic time of fluid
ρ	density of fluid
τ_w	wall shear stress.

Superscript

$+$	dimensionless variables defined by equation (5).

boundary conditions in the thermally developing region can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\rho c_p(\alpha+\varepsilon_H)\frac{\partial T}{\partial r}\right] = \rho c_p u \frac{\partial T}{\partial x} \quad (1)$$

$$T = T_{in} \quad \text{at} \quad x = 0, \quad r \quad (2)$$

$$T = T_{in} \quad \text{at} \quad x > 0, \quad 0 \leqslant r \leqslant R-\Delta \quad (3)$$

$$\rho c_p \alpha \frac{\partial T}{\partial r} = q''_w \quad \text{at} \quad x > 0, \quad r = R \quad (4)$$

where Δ is the thermal boundary layer thickness measured from the wall.

To nondimensionalize the energy equation and boundary conditions, the following dimensionless variables were introduced

$$x^+ = xu^*/\nu, \quad y^+ = yu^*/\nu, \quad u^+ = u/u^*, \quad T^+ = (T-T_{in})\rho c_p u^*/q''_w \quad (5)$$

where $y = (R-r)$ and u^* is the friction velocity. Then, the energy equation and boundary conditions can be expressed in the following nondimensional forms

$$\frac{1}{R^+-y^+}\frac{\partial}{\partial y^+}\left[(R^+-y^+)\left(\frac{1}{Pr}+\frac{\varepsilon_H}{\nu}\right)\frac{\partial T^+}{\partial y^+}\right] = u^+\frac{\partial T^+}{\partial x^+} \quad (6)$$

$$T^+ = 0 \quad \text{at} \quad x^+ = 0, \quad y^+ \quad (7)$$

$$T^+ = 0 \quad \text{at} \quad x^+ > 0, \quad \Delta^+ \leqslant y^+ \leqslant R^+ \quad (8)$$

$$\frac{1}{Pr}\frac{\partial T^+}{\partial y^+} = -1 \quad \text{at} \quad x^+ > 0, \quad y^+ = 0. \quad (9)$$

The above dimensionless energy equation has to be solved to predict the turbulent heat transfer behavior of viscoelastic fluids. The solution of the equation requires explicit expressions for the velocity profile and the eddy diffusivity of heat.

2.2. *Velocity profiles*

The knowledge of the velocity profile in the case of the turbulent pipe flow of viscoelastic fluids is important not only for the solution of the energy equation but it also yields the eddy diffusivity of momentum. Among the many velocity models proposed in the literature the model proposed by Wasan *et al.* [10] which has been successfully applied to Newtonian fluids appears particularly appropriate for extension to the case of viscoelastic fluids.

Wasan *et al.* [10] have proposed compatible expressions for the velocity and continuous eddy diffusivity distributions near a pipe wall, taking into account the continuity and momentum equations. The derived expressions for the dimensionless velocity and the eddy diffusivity of momentum near the wall are

$$u^+ = y^+ + c_1(y^+)^4 + c_2(y^+)^5 \quad \text{where} \quad y^+ < y_c^+ \quad (10)$$

$$\varepsilon_M/\nu = \frac{-4c_1(y^+)^3 - 5c_2(y^+)^4}{1+4c_1(y^+)^3+5c_2(y^+)^4} \quad \text{where} \quad y^+ < y_c^+ \quad (11)$$

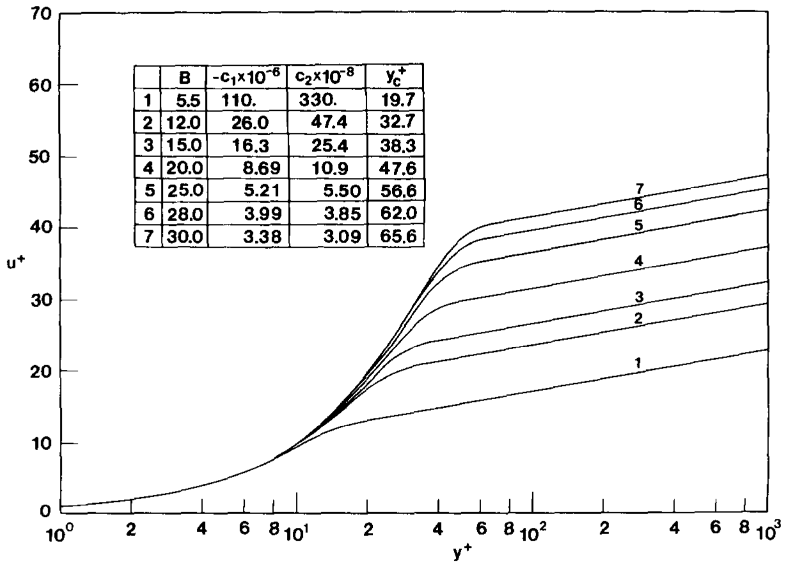

FIG. 1. Velocity profiles for viscoelastic fluids.

where c_1 and c_2 are constants. Equations (10) and (11) yield continuous velocity and momentum eddy diffusivity values in the viscous sublayer and transition region.

Wasan *et al.* then matched the values of the dimensionless velocity and the first and second derivatives of u^+ given by equation (10) with an appropriate expression for the turbulent core, the so-called logarithmic law of the wall

$$u^+ = A \ln y^+ + B \quad \text{where} \quad y^+ > y_c^+. \qquad (12)$$

Then the values of y_c^+, the intersection of the two profiles, as well as c_1 and c_2 can be calculated for given values of A and B. For Newtonian fluids they recommended $c_1 = -104 \times 10^{-6}$, $c_2 = 303 \times 10^{-8}$, and $y_c^+ = 20$.

Subsequently, Edwards and Smith [12] suggested that the velocity model proposed by Wasan *et al.* could be modified to account for the changes brought about in viscoelastic fluids. Experiments [13–17] indicate that the core region turbulent velocity profile in viscoelastic fluids has the same slope (i.e. the same value of A) as in Newtonian fluids but with higher values of the intercept B in equation (12). A fixed value of 2.5 was used for A for viscoelastic fluids, while the value of B was allowed to increase with an increase of elasticity (i.e. an increase in the Weissenberg number). As a result of the increase in elasticity, the velocity profile in the wall region approaches that of the core at increasing values of y_c^+. Values of y_c^+, c_1 and c_2 can be determined for a fixed value of B using the same calculation procedure as outlined for Newtonian fluids.

Values of B corresponding to minimum drag reduction were selected such that they yielded reasonable agreement with the measured friction factor data at Reynolds numbers of 20000 and 30000. To determine the values of B corresponding to the minimum drag asymptote the velocity profiles by Wasan *et al.* were transformed into the equivalent coordinates of the friction factor as a function of the Reynolds number. Then, the values of B were selected as 28 for the Reynolds number of 20000 and as 30 for the Reynolds number of 30000. These values are very close to those predicted by the equation suggested by Edwards and Smith [12]: 26.9 for the Reynolds number of 20000, 29.6 for the Reynolds number of 30000.

Figures 1 and 2 show the results of velocity profiles by Wasan *et al.* for viscoelastic fluids and the corresponding friction factor as a function of the Reynolds number, respectively. The profile identified as 6 in Fig. 1 corresponds to the asymptotic velocity profile for a Reynolds number of 20000, yielding the friction factor result shown on Fig. 2 on the curve, also

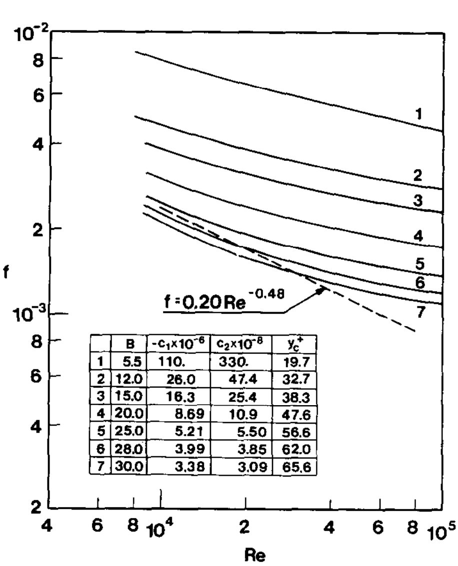

FIG. 2. Relationship of Fanning friction factor and Reynolds number for viscoelastic fluids.

identified as 6. It can be seen that the predicted value of the friction factor is in agreement with the empirical formulation at a Reynolds number of 20 000. Velocity profile 7 and its counterpart friction factor curve yield the asymptotic values for a Reynolds number of 30 000.

2.3. *Relationship of B and the Weissenberg number*

It has been reported that the turbulent velocity profile of viscoelastic fluids is influenced by a number of factors such as the polymer concentration, the level of mechanical degradation, the pipe diameter and flow rate [13–17]. However, according to recent reports [3, 4] the influence of these factors on the fully established dimensionless turbulent pressure drop and heat transfer behavior of viscoelastic fluids are fully accounted for in the Reynolds and Weissenberg numbers.

The value of the Weissenberg number corresponding to the Newtonian velocity profile is zero whereas for a velocity profile corresponding to the minimum drag asymptote the Weissenberg number is equal to or greater than Ws_{cf} [3]. This suggests that each intermediate velocity profile corresponds to an intermediate value of the Weissenberg number lying between $Ws = 0$ and Ws_{cf}. To determine the Weissenberg number corresponding to each velocity profile the results shown on Fig. 2 for the friction factor as a function of Reynolds number were used. This figure reveals a decrease in friction factor with increasing values of B (i.e. increasing elasticity) at a fixed Reynolds number. The corresponding Weissenberg numbers were determined using the empirical correlations for the friction factor as functions of the Weissenberg and Reynolds numbers [18].

Figure 3 shows the values of B which represent the velocity profiles for viscoelastic fluids as functions of the Weissenberg number for Reynolds numbers of 20 000 and 30 000. It is seen that the value of B is a strong function of the Weissenberg number and a weak function of the Reynolds number. At a Reynolds number of 30 000 the limiting value of B is 30, corresponding to a value of Ws_{cf} equal to or greater than 15, while the maximum value of B at a Reynolds number of 20 000 is equal to 28 corresponding to Ws_{cf} equal to or greater than 11. It is noteworthy that the velocity profile by Wasan *et al.* corresponding to the minimum drag asymptote is a function of the Reynolds number whereas the velocity profile for Newtonian fluids is unique.

2.4. *Eddy diffusivity of heat*

The velocity distribution necessary for the solution of the energy equation (6) is now available. The only remaining requirement is an appropriate expression for the eddy diffusivity of heat. As mentioned above the simple Reynolds analogy cannot be applied to viscoelastic fluids. Different empirical relationships for the variation of the eddy diffusivity with the distance from the wall have been used, often expressed in the following form [19–22]

$$\varepsilon_H/\nu = K(y^+)^m. \tag{13}$$

Values of m have been reported in the range of 3–4. For example, Shulman and Pokryvailo [20] in a study of viscoelastic fluids have indicated that $m = 3.0$ allows them to correlate experimental data near the wall region. On the other hand, Son and Hanratty [21] in a mass transfer study of Newtonian fluids have reported that the value of m approaches 4.0 when y^+ decreases to zero. In the current study, equation (13) was used for the eddy diffusivity of heat with the value of 3.0 for m, since the eddy diffusivity of heat for viscoelastic fluids measured by Mizushina and Usui [11] shows better agreement with the power 3.0. The above simple

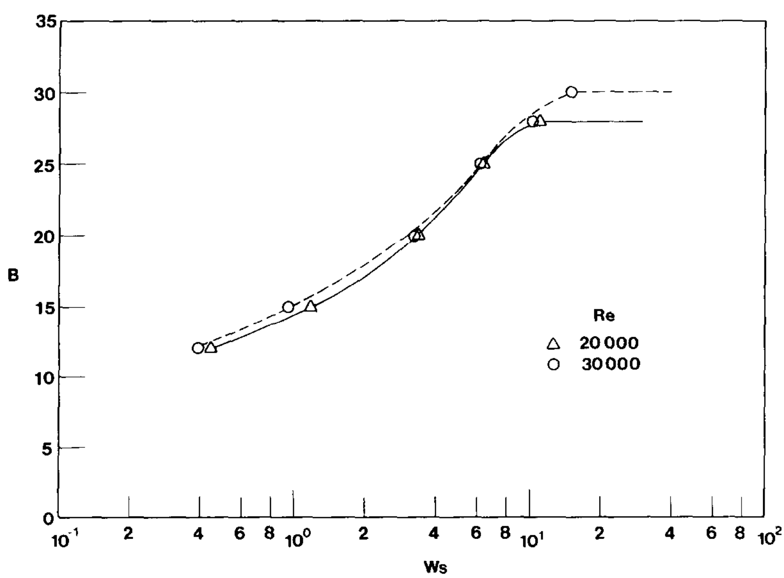

FIG. 3. Relationship of coefficient B and Weissenberg number.

expression for ε_H/ν and the extended Wasan velocity profiles for u^+ given on Fig. 1 were used in equation (6). The successive approximation technique proposed by Cho and Hartnett [22] was used to solve the energy equation. The solution of equation (6) subject to the boundary conditions, equations (7)–(9), evaluated at the location where $\Delta^+ = R^+$ yields the fully established Nusselt value. This was independently checked by solving equation (6) for fully developed thermal conditions.

For each velocity profile the fully established dimensionless heat transfer coefficient, j_H, was calculated as a function of the Reynolds number for different values of the coefficient K in equation (13). The Weissenberg number corresponding to each velocity profile was determined from Fig. 3.

Fortunately extensive experimental studies have yielded empirical correlations for the fully developed dimensionless heat transfer coefficient as functions of the Reynolds and the Weissenberg numbers [18]. The dimensionless heat transfer coefficients resulting from the solution of equation (6) should be the same as the empirically determined values for specified Reynolds and Weissenberg numbers. By trial and error the values of K were determined such that the predicted and measured values were internally consistent.

Figure 4 shows the values of the coefficient K as functions of the Weissenberg number for Reynolds numbers of 20 000 and 30 000 resulting from the procedure. The coefficient K strongly depends on the Weissenberg number and weakly depends on the Reynolds number. The value of K monotonically decreases with increasing Weissenberg number up to the critical Weissenberg number for heat transfer for both Reynolds numbers, and remains constant beyond these values. The eddy diffusivity of heat corresponding to the minimum heat transfer asymptote is much smaller than that for Newtonian fluids and can be expressed by the following equations

$$\varepsilon_H/\nu = 2.0\times10^{-6}(y^+)^3 \quad \text{when} \quad Re_a = 20\,000 \qquad (14)$$

$$\varepsilon_H/\nu = 1.5\times10^{-6}(y^+)^3 \quad \text{when} \quad Re_a = 30\,000. \qquad (15)$$

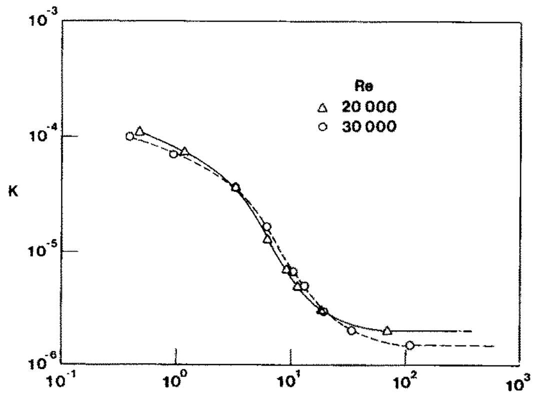

FIG. 4. Relationship of coefficient K and Weissenberg number.

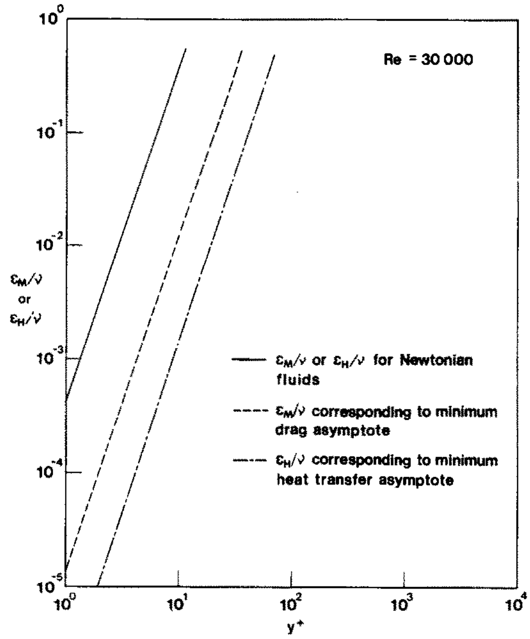

FIG. 5. Eddy diffusivities of momentum and heat corresponding to Newtonian fluids and to minimum asymptotes for a Reynolds number of 30 000.

The corresponding values of the critical Weissenberg number for heat transfer, Ws_{ch}, are 70 for $Re_a = 20\,000$ and 110 for $Re_a = 30\,000$.

2.5. *Turbulent Prandtl number, $\varepsilon_M/\varepsilon_H$*

To evaluate the turbulent Prandtl number of viscoelastic fluids in fully established pipe flow the value of the eddy diffusivity of momentum near the wall was calculated by equation (11). Figure 5 shows the eddy diffusivities of momentum corresponding to a Newtonian fluid and to the minimum drag asymptote as functions of y^+. The eddy diffusivity of momentum for a viscoelastic fluid corresponding to the minimum drag asymptote is smaller than that for a Newtonian fluid by a factor of 30. It is interesting to note that the eddy diffusivity of momentum near the wall is nearly linearly proportional to y^+ with a slope of about 3.0 on a logarithmic graph. For comparison, the eddy diffusivity of heat corresponding to the minimum heat transfer asymptote is also plotted in Fig. 5. This figure clearly shows that the minimum eddy diffusivity of heat is much smaller than the minimum eddy diffusivity of momentum which is consistent with the earlier results reported by the authors [8, 9].

Values of the eddy diffusivities of heat and momentum were calculated at $y^+ = 1$ and 10 and were plotted in Fig. 6 as functions of the Weissenberg number to see the detailed behavior. At fixed values of y^+ the eddy diffusivity of momentum decreases with increasing Weissenberg number up to the critical value for friction, Ws_{cf}. Similarly the eddy diffusivity of heat decreases with increasing Weissenberg number. The major difference is that the eddy diffusivity of heat

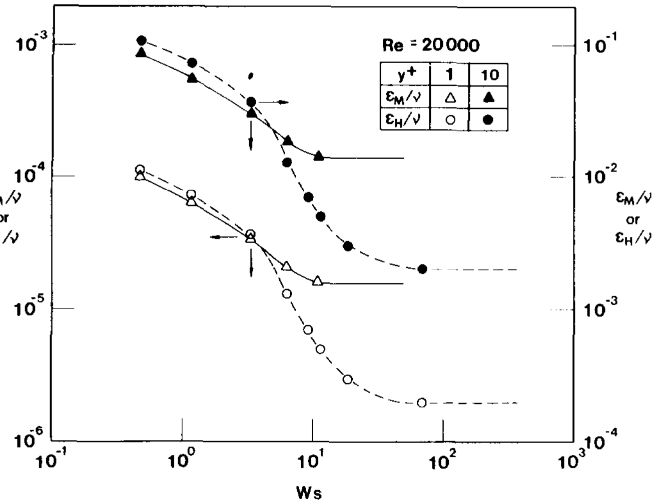

FIG. 6. Weissenberg number effect on eddy diffusivities of momentum and heat evaluated at $y^+ = 1$ and 10 for the Reynolds number of 20 000.

continues to decrease with increasing Weissenberg number beyond the critical Weissenberg number for friction while the eddy diffusivity of momentum remains constant. The eddy diffusivity of heat has a slightly higher value than that of momentum in the Weissenberg number region less than 5 for the specific conditions shown in Fig. 6. However, the eddy diffusivity of heat decreases sharply beyond a Weissenberg number of 5 and becomes smaller than that of momentum.

The turbulent Prandtl number, $\varepsilon_M/\varepsilon_H$, was calculated at two different values of y^+ for the Reynolds number of 20 000 using the results shown in Fig. 6 and was plotted as a function of the Weissenberg number in Fig. 7. The turbulent Prandtl number remains at a constant value slightly lower than unity in the low Weissenberg number region. However, the turbulent Prandtl number starts to increase with increasing Weissenberg number beyond a Weissenberg number equal to approximately 5 and approaches the asymptotic value,

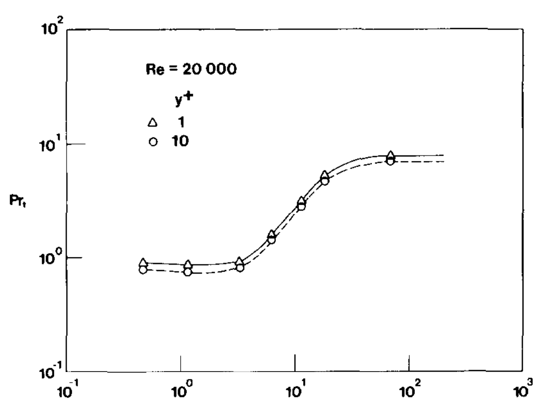

FIG. 7. Weissenberg number effect on turbulent Prandtl number evaluated at $y^+ = 1$ and 10 for the Reynolds number of 20 000.

about 7–8, when the Weissenberg number reaches the critical value for heat transfer, Ws_{ch}. The turbulent Prandtl number for a Reynolds number of 30 000 was also calculated at the same values of y^+ and shows a similar trend as that for the Reynolds number of 20 000. The only difference is that the asymptotic value of the turbulent Prandtl number is higher, 8–9.

An attempt was made by Ng [23] to estimate the turbulent Prandtl number corresponding to the minimum heat transfer asymptote. Ng modified Deissler's model [24] to calculate the velocity and eddy diffusivity of momentum near the wall region corresponding to the minimum drag asymptote. For the core region, Ng reported that his asymptotic friction factor data yielded a logarithmic wall law having a slope of 9.64 rather than a slope of 11.7 proposed by Virk [25]. The eddy diffusivity expression proposed by Reichardt [26] was used in the core region. To calculate the turbulent Prandtl number corresponding to the minimum heat transfer asymptote Ng applied the Reichardt analysis assuming that the ratio of eddy diffusivities is constant across the entire flow field. He found that the predicted heat transfer values agree well with the experiments when the turbulent Prandtl number is 6.7. Notwithstanding the different approach taken by Ng this value is in good agreement with the asymptotic values given on Fig. 7.

3. CONCLUDING REMARKS

Experimental measurements of the friction factors and heat transfer coefficients of viscoelastic fluids in turbulent pipe flow have been used to estimate the eddy diffusivities of momentum and heat. The study reveals that the eddy diffusivities and the turbulent Prandtl number near the wall region are strong functions of the Weissenberg number and weak functions of the Reynolds number. For a fixed value of

y^+ and Reynolds number, the eddy diffusivity of momentum decreases with increasing Weissenberg number (i.e. elasticity of the fluid) up to the critical Weissenberg number for friction. Further increase of the Weissenberg number beyond the critical Weissenberg number for friction does not influence the eddy diffusivity of momentum and it remains at a minimum asymptotic value. The corresponding eddy diffusivity of heat decreases with increasing Weissenberg number beyond the critical Weissenberg number for friction, Ws_{cf}, up to the critical Weissenberg number for heat transfer, Ws_{ch}. Consequently, the turbulent Prandtl number remains relatively constant in the lower Weissenberg number region and increases with increasing Weissenberg number up to a maximum value corresponding to the critical Weissenberg number for heat transfer.

Although these results may be used as a guide for predicting pressure drop and heat transfer behavior of viscoelastic fluids, they provide no insight into the physics of the momentum and energy exchange processes in the presence of elasticity in the fluid. The predicted eddy diffusivity results are, of course, consistent with the experimental observations that the heat transfer decreases more rapidly than the pressure drop as the elasticity increases. But the real questions are why and how the presence of elasticity has a greater effect on the heat transfer process. To gain more insight into the physical mechanisms involved, it may be necessary to recast the corresponding conservation equations to take account of the elastic forces. At the same time, more detailed measurements of the turbulent velocity and temperature profiles including the fluctuating components are essential. Such measurements must be carried out with viscoelastic fluids of known rheological properties.

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ESTIMATION DES DIFFUSIVITES TURBULENTES DE QUANTITE DE MOUVEMENT ET DE CHALEUR POUR LES FLUIDES VISCOELASTIQUES

Résumé—Les diffusivités turbulentes de quantité de mouvement et de chaleur pour un fluide viscoélastique sont évaluées à partir des résultats expérimentaux de chute de pression et de transfert thermique. Les diffusivités turbulentes et le nombre de Prandtl turbulent, près de la région pariétale, sont fortement fonctions du nombre de Weissemberg (une mesure de l'élasticité du fluide) et faiblement fonctions du nombre de Reynolds. Le nombre de Prandtl turbulent $\varepsilon_M/\varepsilon_H$ reste relativement constant pour les faibles valeurs du nombre de Weissemberg et il augmente quand l'élasticité croît, jusqu'à une valeur maximale qui correspond au nombre de Weissemberg critique pour le transfert thermique. La valeur maximale du nombre de Prandtl turbulent dépend faiblement de y^+ et le nombre de Reynolds est de l'ordre de 10.

BESTIMMUNG TURBULENTER AUSTAUSCHGRÖSSEN FÜR IMPULS UND WÄRME BEI VISKOELASTISCHEN FLUIDEN

Zusammenfassung—Es werden die turbulenten Austauschgrößen für Impuls und Wärme für ein viskoelastisches Fluid berechnet unter Verwendung ausführlicher experimenteller Daten von Druckverlust und Wärmeübergang. Es zeigt sich, daß die turbulenten Austauschgrößen für Impuls und Wärme und die turbulente Prandtl-Zahl in der Nähe der Wandregion stark von der Weissenberg-Zahl (einer dimensionslosen Kennzahl für die Fluidelastizität), und schwach von der Reynolds-Zahl abhängen. Die turbulente Prandtl-Zahl, $\varepsilon_M/\varepsilon_H$, ist relativ konstant für kleine Werte der Weissenberg-Zahl und nimmt mit zunehmender Elastizität bis zu einem Maximalwert zu, was der kritischen Weissenberg-Zahl für den Wärmeübergang entspricht. Der maximale Wert der turbulenten Prandtl-Zahl hängt schwach von y^+ und der Reynolds-Zahl ab und liegt in der Größenordnung von 10.

ОЦЕНКА КОЭФФИЦИЕНТОВ ВИХРЕВОЙ ДИФФУЗИИ ИМПУЛЬСА И ТЕПЛА ВЯЗКОУПРУГИХ ЖИДКОСТЕЙ

Аннотация—По измерениям перепада давления и характеристик теплопереноса проведена оценка коэффициентов вихревой диффузии импульса и тепла вязкоупругой жидкости. Найдено, что коэффициенты вихревой диффузии импульса и тепла, а также турбулентное число Прандтля для стенки сильно зависят от числа Вейссенберга (т.е. меры упругости жидкости) и слабо от числа Рейнольдса. Турбулентное число Прандтля, $\varepsilon_M/\varepsilon_H$, почти не изменяется при малых значениях числа Вейссенберга, но возрастает с усилением упругости до максимальной величины, соответствующей критическому значению числа Вейссенберга для теплопереноса. Максимальное значение турбулентного числа Прандтля слабо зависит от y^+ и числа Рейнольдса и, как найдено, равно примерно 10.